LOCAL RESIDENCE TIME, RESIDENCE REVOLUTION AND RESIDENCE VOLUME DISTRIBUTIONS IN TWIN-SCREW EXTRUDERS

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This work was aimed at studying the overall, partial and local residence time distributions (RTD); overall, partial and local residence revolution distributions (RRD) and overall, partial and local residence volume distributions (RVD) in a co-rotating twin screw extruder, on the one hand; and establishing the relationships among them, on the other hand. Emphasis was placed on the effects of the type and geometry of mixing elements (a gear block and various types of kneading elements differing in staggering angle) and process parameters on the RTD, RRD and RVD. Also it was confirmed both experimentally and theoretically that specific throughput, defined as a ratio of throughput ($\dot{Q}$) over screw speed ($N$), controlled all the above three types of residence distributions, be they local, partial or overall. The RRD and RVD do not provide more information on an extrusion process than the corresponding RTD. Rather they are different ways of representing the same phenomena.

Introduction

Twin-screw extruders are one of the most important types of equipment for both food and polymer processing. In the field of polymers, they are mainly used as mixers/reactors for blending, compounding and reactive processing [1]. During an extrusion process, each material element may undergo a different temporal, thermal and/or mechanical history. As a result, its final properties may be different from those of the other material elements [2-5]. The residence time of a material element is the time it spends in the extruder. Since all material elements do not necessarily spend the same elapse of time in the extruder, the residence times have a distribution, called residence time distribution (RTD). The latter provides information not only about the spread of the residence time but also the flow pattern and mixing quality in the extruder. Experimentally the RTD density function, $E(t)$, can be obtained by injecting an inert tracer as a pulse to the extruder and measuring its concentration at a chosen location. The amount of the tracer used should be low enough so as not to disturb the flow field. $E(t)$ can then be calculated according to Eq. 1 [6]:

$$E(t) = \frac{c(t)}{\int_0^\infty c(t)dt} = \frac{c(t)}{\sum_{i=0} c(t)\Delta t} \tag{1}$$

where $c$ is the tracer concentration at time $t$. The mean residence time is defined as

$$\bar{t} = \int_0^\infty tE(t)dt = \frac{\int_0^\infty tc(t)dt}{\int_0^\infty c(t)dt} = \frac{\sum_{i=0} tc(t)\Delta t}{\sum_{i=0} c(t)\Delta t} \tag{2}$$

Besides the above distribution in the residence time which is characterized by $E(t)$, there are also distributions in residence revolution and residence volume, called residence revolution distribution (RRD) and residence volume distribution (RVD), respectively [7]. The RRD and RVD are obtained when the time coordinate of the RTD is converted to the number of accumulated screw revolutions ($n$) and the volume of the extrudate ($v$), respectively. They are given by Eqs. 3 and 4, respectively:

$$F(n) = \frac{c\left(\frac{n}{N}\right)}{\int_0^n c\left(\frac{n}{N}\right)dn} \tag{3}$$
\[ G(v) = \frac{c(v)}{Q} \int_0^\infty \frac{c(v)}{Q} dv \]  

where \( N \) and \( Q \) are the screw speed (revolutions per min or rpm) and material volume throughput (liter/min), respectively. The integration variables of \( F(n) \) and \( G(v) \) are the cumulative screw revolutions \( (n) \) and cumulative extrudate volume \( (v) \), respectively. Their relationships with time are the following:

\[ t = n/N = v/Q \]

According to reference [7], the RVD was a direct measure of the tracer distribution along the screw length, thus a measure of the degree of the axial mixing. On the other hand, the RRD may shed light on the transport behavior of the material in the extruder.

Most previous studies [7-9] dealt with the overall RTD, RRD and RVD, i.e., those of the entire extruder from the hopper to the die exit. Wetzel et al. [10] proposed a method to deconvolute the RTD curves of a model fluid by a predetermined mathematical function. Canevarolo et al. [11] used a direct method to determine the RTD curves of individual elements. However, those studies were limited to the RTD. Potente et al. [12, 13] investigated the local RTD and RVD using a method similar to the one proposed by Wetzel et al. [10]. However, the extruder used was a model twin-screw extruder.

Screw configuration has a pronounced effect on the RTD. Xie et al. [14] analyzed the effects of different screw configurations on the broadness of mixing and compared their subtle differences. Elkouss et al. [15] illustrated a linear relationship between the mean number of screw revolution and the inverse of specific throughput. Specific throughput is defined as the ratio between throughput and screw speed. The linear relationship could differentiate screws composed of different elements. Oberlehner et al. [16] used an off-line method to study the RTD of different types of screw elements such as right- and left-handed ones and mixing ones.

Based on the above mentioned state of the art, this work was aimed at: (1) systematically investigating, at the same time, the overall, partial and local RTD, RRD and RVD in an industrially relevant co-rotating twin-screw extruder with emphasis on the local ones; (2) developing and validating relationships among the above three types of residence distributions; (3) comparing the local RTD, RRD and RVD among different screw elements. This work was made possible by a newly developed in-line measuring instrument [17, 18].

**Experimental**

Experiments were carried out on a co-rotating twin screw extruder with a diameter of 35 mm. Fig. 1 shows the locations of RTD probes and four different screw configurations used in the study. The head of the extruder was equipped with a strip die (L = 30 mm; H = 1.2 mm).
Figure 1. (a) Details of the screw profile of the test zone between probes 1 and 2; (b) three locations (three test points) where the RTD probes were placed; (c) Detailed geometries of the three different types of kneading discs and one type of gear discs used for the test zone. A kneading disc x/y/z had a length of z mm and y discs. The latter were assembled x degrees one with respect to the adjacent one. A gear disc element had two rows of gears along its length of 32 mm. There were 10 gears per row.

Three probes, 1, 2 and 3, were installed along the extruder length. Details about the characteristics and working principles of those probes can be found elsewhere [17, 18]. There were four identical kneading blocks or gear blocks between probes 1 and 2, called the test zone. Three types of kneading blocks (30/7/32, 60/4/32 and 90/5/32) and one type of gear block were chosen to study the effect of the screw configuration on the overall, partial and local RTD. Four screw configurations were used in this work. They differed only in the screw configuration of the test zone. Screw configurations 1, 2 and 3 corresponded to the cases where the test zone was composed of four 30/7/32, 60/4/32 and 90/5/32 kneading blocks, respectively. In screw configuration 4, the test zone was composed of 4 gear blocks. Probe 3 allowed measuring the overall RTD of the extruder between the feeder and the die exit, denoted as $E_3(t)$. Probe 2 measured the partial RTD of the extruder between the feeder and the rear test point of the test zone, denoted as $E_2(t)$. Similarly probe 1 measured the partial RTD of the extruder between the feeder and the front test point of the test zone, denoted as $E_1(t)$. The local RTD density function between probes 1 and 2 will be denoted as $E_{12}(t)$. The screw elements beneath probes 1 and 2 were cylinders. The cylinders ensured full fill of the screw elements beneath probes 1 and 2 and a constant depth between the probes and the screw element surfaces.

Table 1. Experiments carried out in this work and their operating conditions. The barrel temperature was set at 220 °C throughout the screw length.

<table>
<thead>
<tr>
<th>Experimental No.</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
<th>7</th>
<th>8</th>
<th>9</th>
</tr>
</thead>
<tbody>
<tr>
<td>Screw speed N (rpm)</td>
<td>60</td>
<td>90</td>
<td>120</td>
<td>150</td>
<td>120</td>
<td>120</td>
<td>150</td>
<td>150</td>
<td>90</td>
</tr>
<tr>
<td>Mass throughput (kg/h)</td>
<td>10.7</td>
<td>10.7</td>
<td>10.7</td>
<td>10.7</td>
<td>14.3</td>
<td>15.5</td>
<td>13.4</td>
<td>17.8</td>
<td>8.0</td>
</tr>
<tr>
<td>Volume throughput $Q$ ($\times 10^{-3}$ liter/min)</td>
<td>183.8</td>
<td>183.8</td>
<td>183.8</td>
<td>183.8</td>
<td>245.7</td>
<td>266.3</td>
<td>230.2</td>
<td>305.8</td>
<td>137.5</td>
</tr>
<tr>
<td>Specific throughput $Q/N$ ($\times 10^{-3}$ liter/rev.)</td>
<td>3.06</td>
<td>2.04</td>
<td>1.53</td>
<td>1.23</td>
<td>2.04</td>
<td>2.22</td>
<td>1.53</td>
<td>2.04</td>
<td>1.53</td>
</tr>
</tbody>
</table>

Polystyrene (PS) was used as the flow material. Its number and mass average molar masses were $1.23 \times 10^5$ and $2.76 \times 10^5$ g/mole, respectively. The tracer was in the form of masterbatch composed of the PS and anthracene. The concentration of anthracene in the masterbatch was 5% by mass. Those masterbatches were prepared by blending the PS with anthracene in a Haake batch mixer and then extruded into pellets similar to the PS. More details can be found elsewhere [17, 18]. Table 1 shows the experiments carried out in this work and their operating conditions. The volume throughput was calculated by dividing the mass throughput by the melt density of the PS, 970 kg/m$^3$. 

The Polymer Processing Society 23rd Annual Meeting
Results and Discussion

Partial RTD, RRD and RVD

A twin screw extrusion process often works under starved conditions (portions of the extruder are completely filled and others are partially filled). In order to better understand the process, it would be useful to search for a process parameter whose variation would not change the intensity of mixing but the RTD, or vice versa. Early studies [19-24] showed that specific throughput \((Q/N)\) could be one. Fig. 2 shows the RTDs of screw configuration 1 measured at probes 1 and 2 for a given specific throughput. An increase in screw speed with a corresponding increase in throughput shifted the RTD curve to the shorter time domain, as expected. Moreover the RTD curve became narrower. Besides those classical two observations, there was not any obvious correlation or relationship among the RTD curves obtained at different screw speeds and throughputs at a given \(Q/N\). However, when they were normalized with regard to their respective mean residence times, then they all superimposed on single master curves, respectively. Those are shown in Fig. 3 in which the \(E(t)\) versus \(t\) curves are converted to \(E(\tau)\) versus \(\tau\) curves. \(E(t)\) is the dimensionless residence time distribution density function and \(\tau\) is the dimensionless residence time \((\tau = t/\bar{t})\) [23, 24]. This confirms the results of the literature that the dimensionless RTD density function \(E(\tau)\) versus \(\tau\) curve was unique when \(Q/N\) and screw configuration were fixed, irrespective of \(Q\) and \(N\).

![Figure 2](image1.png)

**Figure 2.** Effect of increasing screw speed and throughput on the RTD for a \(Q/N\) of \(1.53 \times 10^{-3}\) liter/revolution. (a) Probe 1; (b) probe 2. Screw configuration 1.

![Figure 3](image2.png)

**Figure 3.** Dimensionless residence time distribution density function \(E(\tau)\) versus \(\tau\) curves corresponding to the \(E(t)\) versus \(t\) curves in Fig. 2. (a) Probe 1; (b) probe 2. Screw configuration 1, \(Q/N = 1.53 \times 10^{-3}\) liter/revolution. Note that all the \(E(\tau)\) versus \(\tau\) curves fall on a single curve.

When the RTD curves in Fig. 2 were converted to the screw revolution and extrudate volume coordinates using Eqs. 3 and 4, respectively, two pairs of master curves were obtained. They are shown in Figs. 4 and 5, respectively. This indicates that the RRD and RVD were also unique when \(Q/N\) and screw configuration were fixed, irrespective of \(Q\) and \(N\).

The key question is then what are the inherent factors that led to the master curves of the \(E(\tau)\) versus \(\tau\), RRD versus \(n\) and RVD versus \(v\) shown in Figs. 3-5, respectively? What are the relationships among the \(E(\tau)\) versus \(\tau\), RRD versus \(n\) and RVD versus \(v\)? Actually all of them are derived from the \(E(t)\) versus \(t\) and their respective abscissas are as follows:

\[
\tau = t/\bar{t} \quad (6)
\]

\[
n = tN \quad (7)
\]
According to Eq. 5, one has the following equality:

\[ \tau = \frac{t}{iN} = \frac{v}{iQ} \]  

(9)

**Figure 4.** RRD corresponding to probes 1 and 2, respectively. (a) Probe 1; (b) probe 2; screw configuration 1; \( Q/N = 1.53 \times 10^{-3} \) liter/revolution. Note that all the RRD curves overlap.

**Figure 5.** RVD corresponding to probes 1 and 2, respectively. (a) Probe 1; (b) probe 2; screw configuration 1; \( Q/N = 1.53 \times 10^{-3} \) liter/revolution. Note that all the RVD curves overlap.

Considering Eq. 9 and the fact that when \( Q/N \) is fixed, \( E(\tau) \) versus \( \tau \), RRD versus \( n \) and RVD versus \( v \) are master curves and are independent of \( Q \) and \( N \), one arrives at the following relationships for a given \( Q/N \):

\[ NT = k_1 = \text{constant}1 \]  

(10)

\[ Q\bar{\tau} = k_2 = \text{constant}2 \]  

(11)

where \( k_1 \) and \( k_2 \) are constants and \( k_2/k_1 = Q/N \). In other words, when \( Q/N \) is fixed, \( NT \) and \( Q\bar{\tau} \) should be constants, whatever the values of \( Q \) and \( N \). This will be blockussed later.

The mean residence time can be determined by:

\[ \bar{\tau} = \frac{V \times f}{Q} \]  

(12)

where \( V \) is the free volume in the extruder barrel and \( f \) is the mean degree of fill. When Eq. 12 is introduced to Eqs. 10 and 11, respectively, the following equation is obtained:

\[ f = k_1 \frac{Q}{N} = k_2 \frac{Q}{V} = \text{constant}3 \]  

(13)

Eq. 13 indicates that when \( Q/N \) is fixed, the mean degree of fill is fixed, regardless of \( Q \) and \( N \). Mudalamane et al. [25] proposed the following equation to describe the relationship between the complete fill length, \( FL \), and \( Q/N \):

\[ FL = K_n \left( \frac{Q}{N} \right) \left( \frac{Q}{K_{\text{pump}} Q/N} \right) \]  

(14)
where $K_r$ is a constant that only depends on screw geometry and $K_{pump}$ is a function of screw design. This equation shows that for given screw geometry and screw design, once $Q/N$ is fixed the complete fill length is fixed.

### Table 2. Values of $\tilde{Q}_t$ corresponding to a given $Q/N$ for four screw configurations in Fig.1 at probes 1 and 2.

<table>
<thead>
<tr>
<th>Probe</th>
<th>$\tilde{T}_{Q_1}$ (liter)</th>
<th>$\tilde{T}_{Q_2}$ (liter)</th>
<th>$\tilde{T}_{Q_3}$ (liter)</th>
</tr>
</thead>
<tbody>
<tr>
<td>4 x 30° kneading discs</td>
<td>0.29</td>
<td>0.30</td>
<td>0.29</td>
</tr>
<tr>
<td>4 x 60° kneading discs</td>
<td>0.30</td>
<td>0.31</td>
<td>0.31</td>
</tr>
<tr>
<td>4 x 90° kneading discs</td>
<td>0.32</td>
<td>0.34</td>
<td>0.32</td>
</tr>
<tr>
<td>4 x gear discs</td>
<td>0.31</td>
<td>0.29</td>
<td>0.33</td>
</tr>
<tr>
<td>Probe 1</td>
<td>0.34</td>
<td>0.37</td>
<td>0.32</td>
</tr>
<tr>
<td>Probe 2</td>
<td>0.35</td>
<td>0.37</td>
<td>0.36</td>
</tr>
</tbody>
</table>

Note: $Q/N = 2.04 \times 10^{-3}$ liter/revolution with mass throughputs of $Q_1$, $Q_2$ and $Q_3$ being 10.7, 14.3 and 17.8 kg/h and the corresponding screw speeds, $N_1$, $N_2$ and $N_3$ being 90, 120 and 150 rpm, respectively. $\tilde{T}_1$, $\tilde{T}_2$ and $\tilde{T}_3$ were the mean residence times of the three pairs of $Q$ and $N$, respectively.

According to Eqs. 10 and 11, for given screw profile and $Q/N$, the value of $\tilde{N}$ and that of $\tilde{Q}$ should be constant, respectively, regardless of $N$ or $Q$. This is confirmed by the data shown in Table 2 corresponding to the values of $\tilde{Q}$ obtained at probes 1 and 2 for the four different screw configurations in Fig. 1 and for a $Q/N$ of $2.04 \times 10^{-3}$ liter/revolution with three pairs of $Q$ and $N$ values. When $Q/N$, screw configuration and probe location are fixed, the values of $\tilde{Q}$ for all the three different values of $Q$ are indeed the same within the experimental and numerical calculation errors. For example, the value of $\tilde{Q}$ was between 0.42 and 0.44 for the 4 gear block configuration and probe 2, regardless of the values of $Q$ and $N$.

**Local RTD, RRD and RVD**

The flow and mixing conditions at the two boundaries of the test zone of this work were expected to be such that the statistical theory for the RTD would apply. Detailed blockusssions on the validity of that theory can be found elsewhere [26-28]. That theory stipulates that for a closed system composed of two statistically independent elements, A and B, the overall RTD density function $E(t)$ is related to those of the two elements, $E_A(t)$ and $E_B(t)$, by the following equation:

$$E(t) = \int_0^\infty E_A(t) \cdot E_B(t-\tau) d\tau$$

This equation shows that knowing any two of the three RTD density functions allows calculating the third one either by convolution or deconvolution. However, it should be pointed that when it comes to deconvoluting experimental data, numerical difficulties often rise because of data scattering. To get rid of numerical difficulties, we first fit the experimental data with an appropriate mathematical function. We used the following expression to fit our RTD curves [29]:

$$E(t) = a \cdot t^{c-1} \cdot e^{-b \cdot c \cdot t^c} \cdot \exp\left(\frac{(b \cdot c \cdot t^c - 1) \cdot \left(\frac{e^{-c \cdot t^c}}{c}\right)}{c}\right)$$

where $a$, $b$ and $c$ are adjustable parameters. They were obtained by fitting the experimental data using a least squared optimization method. Then the fitted curve was normalized, assuring that $\int_0^\infty E(t) dt = 1$.

**Figure 6.** Effects of the screw speed (a) and throughput (b) on the local RTD of the test zone of screw configuration 1.
The local RTD curves in the test zone between probes 1 and 2, $E_{12}(t)$, were obtained by the deconvolution of the experimental data of $E_1(t)$ and $E_2(t)$. The method used for the deconvolution can be found elsewhere [17].

Fig. 6 shows the effects of screw speed and throughput on $E_{12}(t)$ for screw configuration 1. Increasing screw speed with throughput being held constant shifted the local RTD curve to the short time domain. However, its shape did not change much (Figure 6a). Increasing throughput with screw speed being held constant also shifted the RTD curve to the short time domain. Moreover, its width became shaper. Therefore, the shape of the RTD was much more affected by throughput than screw speed.

Fig. 7 shows the local RTD, RRD and RVD curves between probes 1 and 2 for given values of Q/N. Again they all fall on a single curve, which is consistent with the overlapping of the partial RRD and RVD curves in Figs 4 and 5.

Figure 7. Local RTD (a), RRD (b) and RVD (c) curves between probes 1 and 2 for screw configuration 3 at a given Q/N ($2.04 \times 10^{-3}$ liter/revolution). They are obtained by deconvolution. Note that all the local RTD, RRD and RVD curves fall on single master curves, respectively.

Fig. 8 shows the effect of screw configuration on the local RTD, RRD and RVD curves of the test zone between probes 1 and 2. They were obtained by deconvolution of the data obtained by probes 1 and 2 using Eq.15. It is seen that both the mean residence time and the axial mixing quality characterized by the width of the RTD, followed the order: $30^\circ < 60^\circ << 90^\circ <$ gear block.
The RRD and RVD curves followed the same trend as the RTD, as expected. When the operating conditions were fixed, an increase in the staggering angle of the kneading block from 30° to 90° shifted the local RRD and RVD to higher screw revolution and the larger extrudate volume domains, respectively. The shift with the gear blocks was the most significant, indicating that conveying a given amount of tracer from probe 1 to probe 2 needed more screw revolutions or more extrudate volume when the gear blocks were used.

Conclusion

The overall and partial RTD were measured directly during the extrusion process using a new in-line RTD measurement instrument that was developed in a previous study. The local RTD were calculated by deconvolution based on a statistical theory. We studied the overall, partial and local residence time distributions (RTD); overall, partial and local residence revolution distributions (RRD) and overall, partial and local residence volume distributions (RVD) in a co-rotating twin screw extruder and established the relationships among them. It was confirmed theoretically and experimentally that specific throughput $Q/N$, defined as a ratio of throughput ($Q$) over screw speed ($N$), was indeed a key process parameter for controlling all the above three types of residence distributions. For a given value of $Q/N$, the overall, partial and local RTD were different when $Q$ and $N$ varied. However the corresponding dimensionless RTD as well as the RRD and RVD all fell on single master curves, respectively. This is because the mean degree of fill and complete fill length were the same for a given value of $Q/N$. This indicates that the RRD and RVD, be they overall, partial or local, do not provide more information on an extrusion process than the corresponding RTD. Rather they offer different ways of representing the same phenomena.

Comparison of the local RTD for a local zone composed of 4 identical gear blocks or 4 identical kneading blocks with a staggering angle of 30°, 60° or 90° led to the conclusion that the delay time, the mean residence time and mixing performance followed the order: 30° < 60° << 90°< gear blocks.

Acknowledgements

The authors thank the National Natural Science Foundation of China through grant numbers 50390097 and 20310285, the Ministry of Science and Technology of China through an international cooperation program (grant number: 2001CB711203) and the Association Franco-Chinoise pour la Recherche Scientifique et Technique - AFCRST) through grant number: PRA Mx02-07) for their financial support.

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